Mechanics

Chapter 1: Projectile (Practice)

Solutions

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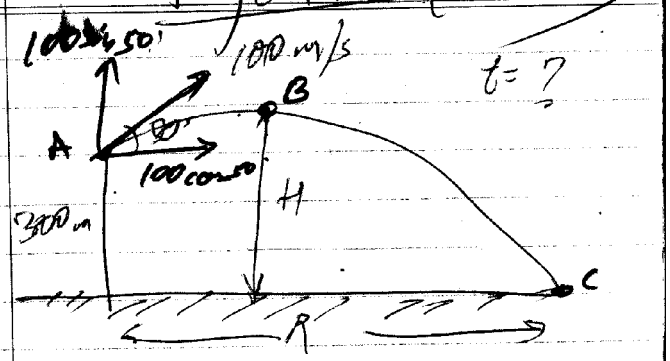
Navlakhi's

Projectile (Rocket)

(1) 100 m/s, 50°



t = ?

(a) A - B.

Vertical

$s = x = (H - 300)$

$v = 0$

$u = 100 \sin 50°$

$v^2 = u^2 - 2gs$

$0 = (100 \sin 50)^2 - 2 \times 9.81 \times (H - 300)$

$\therefore H = 599.09\ m$

(b) A - C

Vertical

$s = -300\ M$

$u = 100 \sin 50°$

$s = ut - \frac{1}{2} g t^2$

$-300 = 100 \sin 50\, t - 0.5 \times 9.81 \times t^2$

$\therefore t = 18.86\ s$

(c) A-C

Horizontal

$s = R$

$u = 100\cos 50°$

$t = 18.86\ s$

$s = ut$

$= (100\cos 50)(18.86)$

$= 1212.30\ m.$

(d) A-C

Vertical

$u = 100\sin 50°$

$v = ?$

$t = 18.86\ s$

$v = u - gt$

$= 100\sin 50 - 9.81(18.86)$

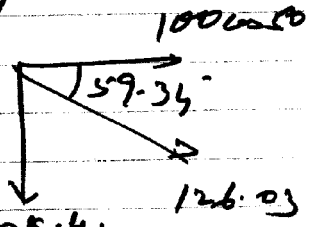
$= -108.41\ m/s$

$v = \sqrt{(108.41)^2 + (100\cos 50)^2}$

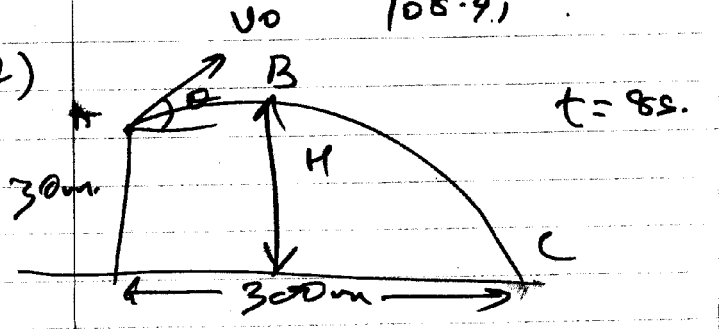
$v = 126.03\ m/s$

$\theta = \tan^{-1}\left(\frac{108.41}{100\cos 50}\right)$

$= 59.34°$



(2)



$t = 8s.$

A-C

Horizontal

$s = 300m$

$u = v_0 \cos\theta$

$t = 8s$

$s = ut$

$\frac{300}{8} = v_0 \cos\theta$ — (1)

Vertical

$s = -30m$

$u = v_0 \sin\theta$

$t = 8s$

$s = ut - \frac{1}{2}gt^2$

$-30 = v_0 \sin\theta (8) - \frac{1}{2} \times 9.81 (8)^2$

$v_0 \sin\theta = 35.49$ — (2)

(1) ÷ (2)

$\tan\theta = \frac{300}{8 \times 35.49}$

$\therefore \theta = 43.42$

Sub in (1) 51.63

$v_0 = 54.56 m/s$

B-C

$u = 54.56 \sin 43.42°$

$v = 0$

$v^2 = u^2 - 2gs$

$0 = (54.56 \sin 43.42)^2 - 2 \times 9.81 (H - 30)$

$\therefore H = 94.19 m$

(3) A–B

Horizontal

$s = 8\text{ m}$

$u = V_0 \cos 30°$

$s = ut$

$\therefore t = \dfrac{8}{V_0 \cos 30°}$

Vertical

$s = -3.5\text{ m}$

$u = V_0 \sin 30°$

$s = ut - \frac{1}{2}gt^2$

$-3.5 = V_0 \sin 30 \left(\dfrac{8}{V_0 \cos 30}\right) - \frac{1}{2} g \left(\dfrac{8}{V_0 \cos 30}\right)^2$

$-3.5 = 8 \tan 30 - \frac{1}{2} 9.81 \left(\dfrac{8}{V_0 \cos 30}\right)^2$

$\therefore V_0 = 7.18\text{ m/s}$

Vertical

$u = 7.18 \sin 30°$

$v = ?$

$t = 1.29\text{ s}$

$v = u - gt$

$= 7.18 \sin 30 - 9.81 \times 1.29$

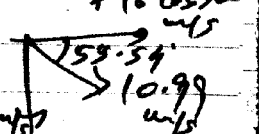
$= -9.06\text{ m/s}$

$v = \sqrt{9.06^2 + (7.18 \cos 30°)^2}$

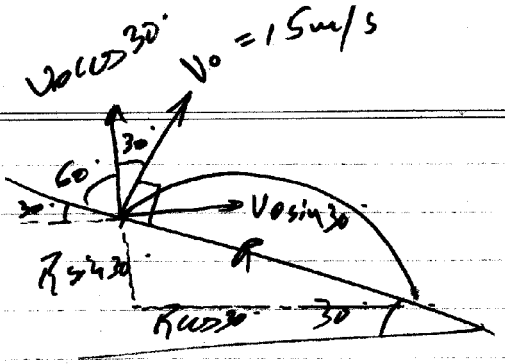
$= 10.99\text{ m/s}$

$\theta = \tan^{-1}\left(\dfrac{9.06}{7.18 \cos 30}\right)$

$= 55.54°$



(4)



Horizontal

$s = R\cos 30°$

$u = V_0 \sin 30°$

$t = \dfrac{R\cos 30°}{15 \sin 30°}$

Vertical

$s = -R\sin 30°$

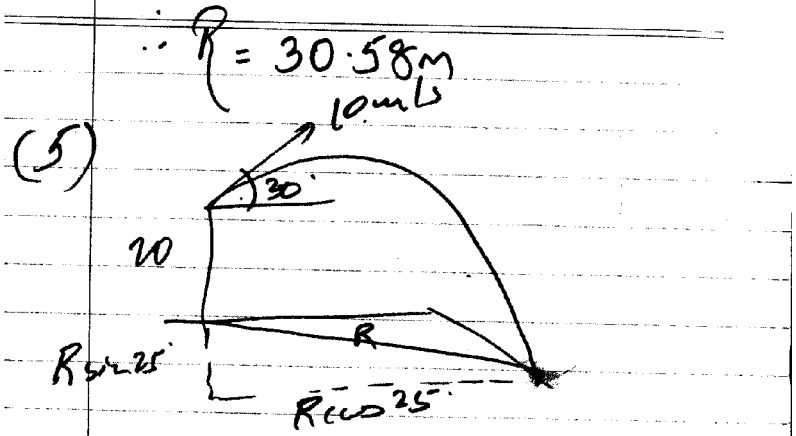
$u = V_0 \cos 30°$

$s = ut - \frac{1}{2}gt^2$

$-R\sin 30° = 15\cos 30°\left(\dfrac{R\cos 30°}{15\sin 30°}\right) - \frac{1}{2}g\left(\dfrac{R\cos 30°}{15\sin 30°}\right)^2$

$\therefore R = 30.58$ m

(5)



Horizontal

$s = R\cos 25°$

$u = 10\cos 30°$

$\therefore t = \dfrac{R\cos 25}{10\cos 30°}$

Vertical

$s = -20 - R\sin 25°$

$u = 10 \sin 30°$

$s = ut - \frac{1}{2}gt^2$

$-20 - R\sin 25° = 10\sin 30°\left(\dfrac{R\cos 25°}{10\cos 30°}\right) - \frac{1}{2}g\left(\dfrac{R\cos 25}{10\cos 30}\right)^2$

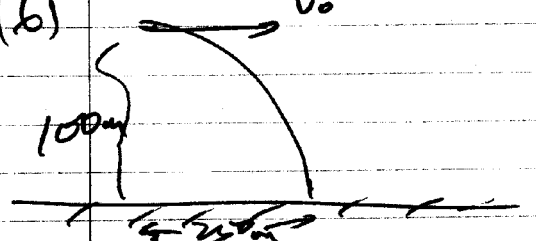
$R = 30.01\text{ m}$

Sub in horizontal

$t = \frac{30.01 \cos 25}{70 \cos 30^\circ}$

$= 3.14\text{ s}.$

(6)



Horizontal

$s = 250\text{ m}$

$u = v_0$

$\therefore t = \frac{250}{v_0}$ — (1)

Vertical

$s = -100\text{ m}$

$u = 0$

$s = ut - \frac{1}{2}gt^2$

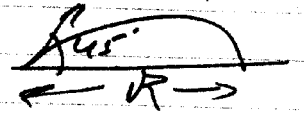
$-100 = 0 - \frac{1}{2} \times 9.81 \times t^2$

$\therefore t = 4.52\text{ s}.$

Sub in (1)

$\therefore v_0 = 55.31\text{ m/s}$

(7) $\theta = 45^\circ$



Horizontal

$s = R$

$u = 260 \cos 45^\circ$

$\therefore t = \frac{R}{260 \cos 45^\circ}$

Vertical

$s = 0$

$u = 260 \sin 45^\circ$

$\therefore s = ut - \frac{1}{2}gt^2$

$0 = 260 \sin 45^\circ - \frac{1}{2}g\left(\frac{R}{260\cos 45^\circ}\right)$

$\therefore R = 6890.92\ m$

(ii) Horizontal

$s = 12$

$u = 15\cos\alpha$

$\therefore t = \frac{12}{15\cos\alpha}$

Vertical

$s = h - 1.3$

$u = 15\sin\alpha$

$s = ut - \frac{1}{2}gt^2$

$h - 1.3 = 15\sin\alpha\left(\frac{12}{15\cos\alpha}\right) - \frac{1}{2}g\left(\frac{12}{15\cos\alpha}\right)^2$ → (1)

$h - 1.3 = 12\tan\alpha - \frac{1}{2}g\frac{144}{225}\sec^2\alpha$

Diff w.r.t. α

$\frac{dh}{d\alpha} = 12\sec^2\alpha - \frac{1}{2}g\left(\frac{144}{225}\right)2\sec^2\alpha\tan\alpha$

Put $dh/d\alpha = 0$

$\therefore \tan\alpha = 1.911$

$\alpha = 62.38^\circ$

Put in (1)

$\therefore h = 9.63\ m$

(9) Horizontal

$s = 12$

$u = 15\cos\alpha$

$t = \frac{12}{15\cos\alpha}$

Vertical

$s = 7.5 - 1.3 = 6.2\text{m}$

$u = 15\sin\alpha$

$s = ut - \frac{1}{2}gt^2$

$6.2 = 15\sin\alpha\left(\frac{12}{15\cos\alpha}\right) - \frac{1}{2}g\left(\frac{12}{15\cos\alpha}\right)^2$

$\therefore \alpha = 47.41°$ & $69.91°$

(10) (i) Horizontal

$s = 3\text{m}$

$u = V_0\cos 30°$

$\therefore t = \frac{3}{V_0\cos 30°}$

Vertical

$s = -2.5\text{m}$

$u = -V_0\sin 30°$

$s = ut - \frac{1}{2}gt^2$

$-2.5 = -V_0\sin 30°\left(\frac{3}{V_0\cos 30°}\right) - \frac{1}{2}g\left(\frac{3}{V_0\cos 30°}\right)^2$

$\therefore \boxed{V_0 = 8.75\text{ m/s}}$

(ii) Vertical

$s = -3.3\text{m}$

$u = -V_0\sin 30°$

$s = ut - \frac{1}{2}gt^2$

$-3.3 = -V_0\sin 30°\left(\frac{3}{V_0\cos 30°}\right) - \frac{1}{2}g\left(\frac{3}{V_0\cos 30°}\right)^2$

$V_0 = 6.13\text{m/s}$

$6.13 \text{ m/s} \leq v_0 \leq 8.75 \text{ m/s}$

(ii) Horizontal

$s = 9 \text{ m}$

$u = v_0 \cos 25°$

$\therefore t = \frac{9}{v_0 \cos 25°}$

Vertical

$s = -5 \text{ m}$

$u = v_0 \sin 25°$

$s = ut - \frac{1}{2}gt^2$

$-5 = v_0 \sin 25° \left(\frac{9}{v_0 \cos 25°}\right) - \frac{1}{2}g\left(\frac{9}{v_0 \cos 25°}\right)^2$

$\therefore v_0 = 7.25 \text{ m/s}$

Vertical

$u = 7.25 \sin 25°$

$t = \frac{9}{7.25 \cos 25°}$

$v = u - gt$

$= 7.25 \sin 25° - 9.81\left(\frac{9}{7.25 \cos 25°}\right)$

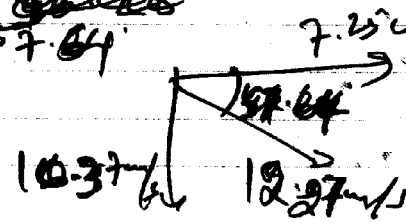
$= -10.37 \text{ m/s}$

$v = \sqrt{10.37^2 + (7.25 \cos 25°)^2}$

$= 12.27 \text{ m/s}$

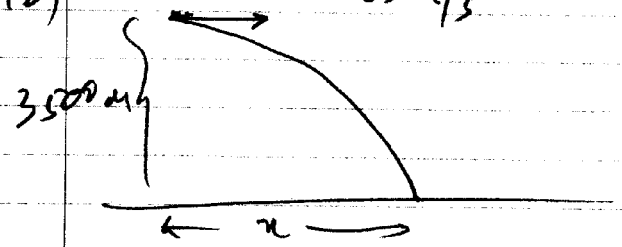
$\theta = \tan^{-1}\left(\frac{10.37}{7.25 \cos 25°}\right)$

$= 57.64°$



(12)

$(324-90)\times 5/18$
$= 65 m/s$

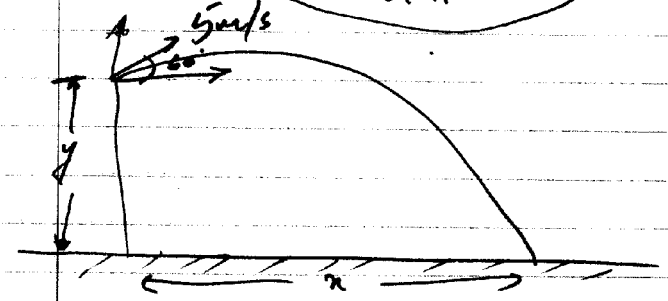


Horizontal
$s = x$
$u = 65$
$\therefore t = \frac{x}{65}$

Vertical
$s = -3500m$
$u = 0$
$\therefore s = ut - \frac{1}{2}gt^2$
$-3500 = -\frac{1}{2}g\left(\frac{x}{65}\right)^2$
$\therefore x = 1736.31 m$

(13)

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$



Horizontal
$t = \frac{x}{5\cos 60°}$

Vertical
$s = -y$
$u = 5\sin 60°$
$s = ut - \frac{1}{2}gt^2$

$$-y = 5\sin 60^\circ\left(\frac{x}{5\cos 60^\circ}\right) - \frac{1}{2}g\left(\frac{x}{5\cos 60^\circ}\right)^2$$

$$y = -x\tan 60^\circ + 0.7848x^2$$

$$\frac{dy}{dx} = -\tan 60^\circ + 2(0.7848)x$$

$$\frac{d^2y}{dx^2} = 2(0.7848)$$

(i) $\left.\frac{dy}{dx}\right|_{t=0} = -\tan 60^\circ$ at $t=0$, $x=0$

$$\frac{d^2y}{dx^2} = 2(0.7848)$$

$$\therefore f = 5.097\ m.$$

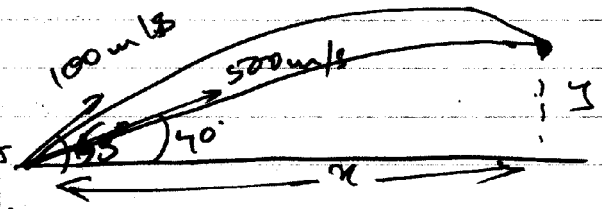
(ii) $\left.\frac{dy}{dx}\right|_{t=0.5} = -\tan 60^\circ + 2(0.7848)x\ (5\cos 60^\circ \times 0.5)$

$$= 0.2299$$

$$\frac{d^2y}{dx^2} = 2(0.7848)$$

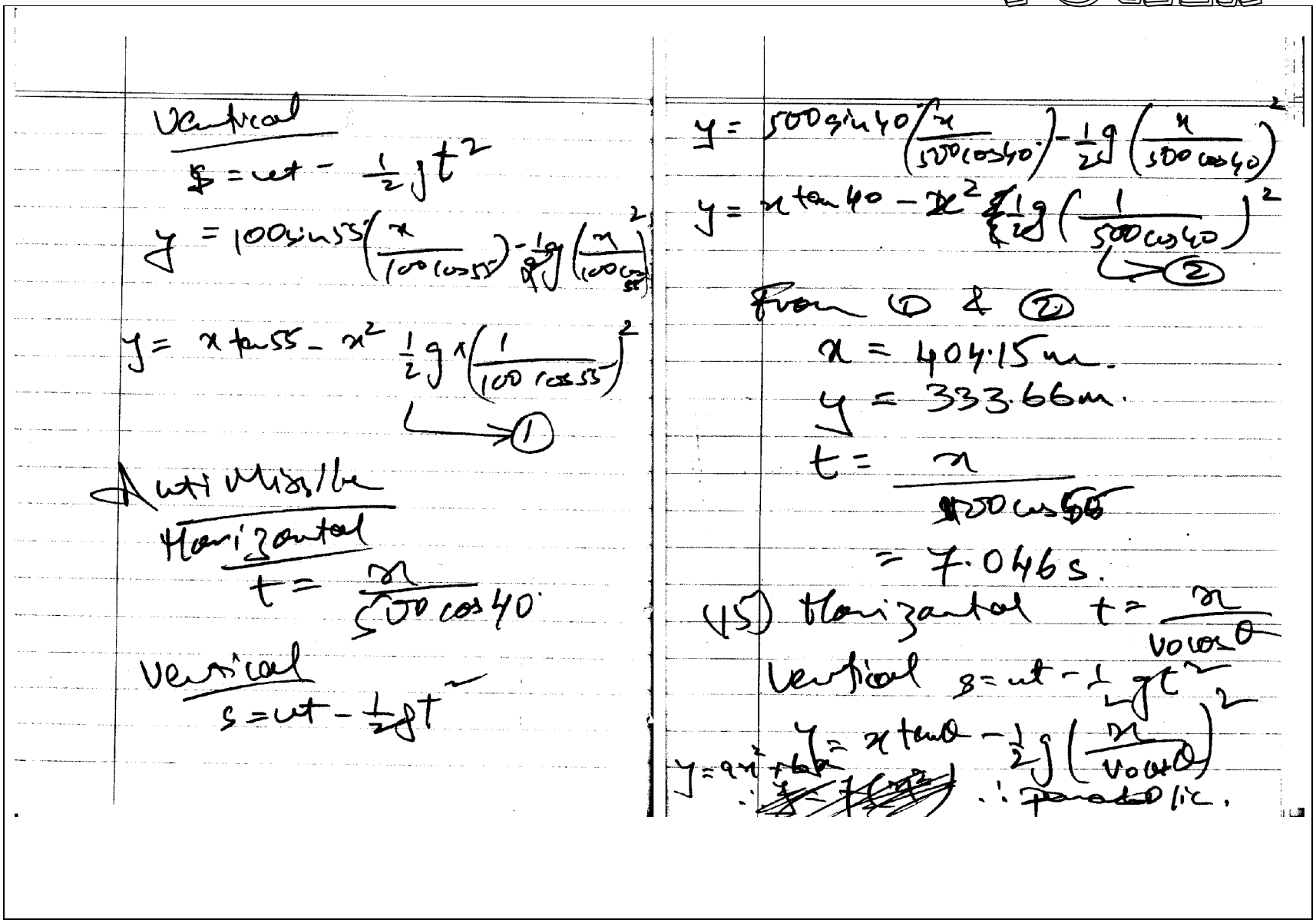
$$f = 0.6883\ m.$$

(14)



Missile Horizontal

$$t = \frac{x}{100\cos 53^\circ}$$



Vertical

$s = ut - \frac{1}{2}gt^2$

$y = 100\sin 55^\circ\left(\frac{x}{100\cos 55^\circ}\right) - \frac{1}{2}g\left(\frac{x}{100\cos 55^\circ}\right)^2$

$y = x\tan 55 - x^2\,\frac{1}{2}g\left(\frac{1}{100\cos 55}\right)^2$ → (1)

Anti Missile

Horizontal

$t = \frac{x}{500\cos 40^\circ}$

Vertical

$s = ut - \frac{1}{2}gt^2$

$y = 500\sin 40^\circ\left(\frac{x}{500\cos 40^\circ}\right) - \frac{1}{2}g\left(\frac{x}{500\cos 40^\circ}\right)^2$

$y = x\tan 40 - x^2\,\frac{1}{2}g\left(\frac{1}{500\cos 40^\circ}\right)^2$ → (2)

From (1) & (2)

$x = 404.15\text{ m}$.

$y = 333.66\text{ m}$.

$t = \frac{x}{500\cos 40^\circ}$

$= 7.046\text{ s}$.

(45) Horizontal $t = \frac{x}{v_0\cos\theta}$

Vertical $s = ut - \frac{1}{2}gt^2$

$y = x\tan\theta - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2$

$y = ax^2 + bx$

$\therefore$ parabolic.

(16) $t = 12s$.

$V_H = V_{final} \cos 60°$

$V_V = V_{final} \sin 60°$

Vertical

$V = u - gt$

$-V_{final} \sin 60° = 0 - 9.81(12)$

$\therefore V_{final} = 135.93 m/s$

$\therefore V_H = 135.93 \cos 60°$

$= 67.97 m/s$

Horizontal

$s = 67.97(12)$

$= 815.64 m$

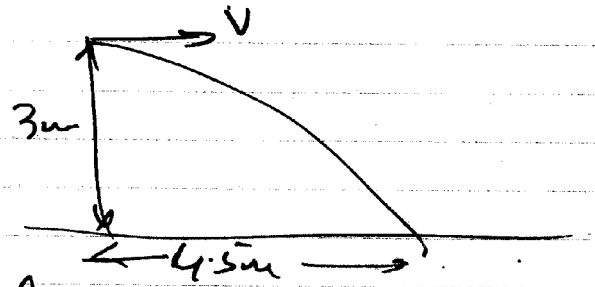
Vertical

$s = ut - \frac{1}{2}gt^2$

$h = 0 - \frac{1}{2} \times 9.81 (12)^2$

$= -706.32 m.$

(17)



Horizontal

$t = \frac{4.5}{V}$

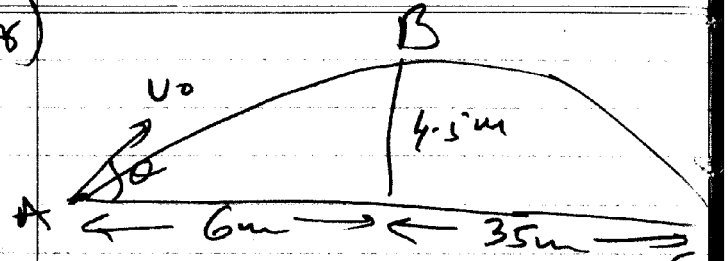
Vertical

$s = ut - \frac{1}{2}gt^2$

$-3 = 0 - \frac{1}{2} \times 9.81 \left(\frac{4.5}{V}\right)^2$

$\therefore V = 5.75 m/s$

(18)



A-C:

Horizontal

$$t = \frac{41}{v_0 \cos\theta}$$

Vertical

$$s = ut - \frac{1}{2}gt^2$$

$$0 = v_0 \sin\theta - \frac{1}{2}g\left(\frac{41}{v_0\cos\theta}\right)$$

$$v_0^2 \sin\theta\cos\theta = \frac{1}{2}g(41)$$

$$v_0^2 = \frac{\frac{1}{2}g(41)}{\sin\theta\cos\theta} \quad \text{---} (1)$$

A-B

Horizontal

$$t = \frac{6}{v_0\cos\theta}$$

Vertical

$$s = ut - \frac{1}{2}gt^2$$

$$4.5 = v_0\sin\theta\left(\frac{6}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{6}{v_0\cos\theta}\right)^2$$

$$\therefore 4.5 = 6\tan\theta - \frac{1}{2}g\,36\left(\frac{1}{v_0\cos\theta}\right)^2$$

$$4.5 = 6\tan\theta - \frac{1}{2}g\,36\,\frac{1}{\cos^2\theta}\,\frac{\sin\theta\cos\theta}{\frac{1}{2}g(41)}$$

$$4.5 = 6\tan\theta - \frac{36}{41}\tan\theta$$

$$\therefore \theta = 41.3°$$

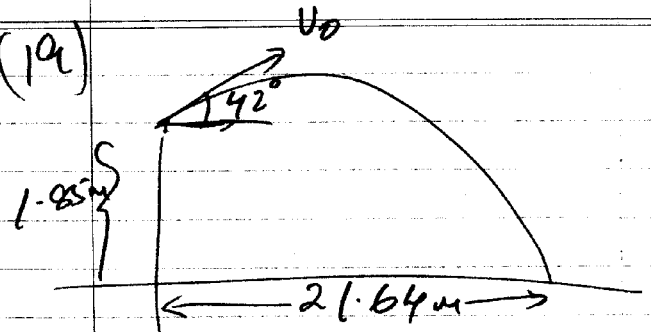
sub in (1)

$$\therefore v_0 = \sqrt{405.59}\ \text{m/s}$$

$$= 20.14\ \text{m/s}.$$

$m = 6kg$

(19)



(a) Horizontal

$$t = \frac{21.64}{V_0 \cos 42^\circ} \quad \text{—(1)}$$

Vertical

$$s = ut - \frac{1}{2}gt^2$$

$$-1.85 = V_0 \sin 42 \left(\frac{21.64}{V_0 \cos 42}\right) - \frac{1}{2}g\left(\frac{21.64}{V_0 \cos 42}\right)^2$$

$\therefore V_0 = 13.96 \text{ m/s}$

(b) Sub in (1)

$t = 2.09s$.

(c) Vertical

$v = u - gt$

$= V_0 \sin 42 - 9.81(2.09)$

$= -11.16$

$V_{final} = \sqrt{11.16^2 + (13.96 \cos 42)^2}$

$= 15.24 \text{ m/s}$

Initial KE $= \frac{1}{2} \times 6 \times (13.96)^2$

$= 584.164 J$

Final KE $= \frac{1}{2} \times 6 \times (15.24)^2$

$= 696.77 J$

(20) (a) $\theta = 45°$

Vertical

$s = ut - \frac{1}{2}gt^2$

$0 = 25 \sin 45 - \frac{1}{2}gt$

$\therefore t = 3.6\,s$

Horizontal

$s = ut$

$= 25 \cos 45 (3.6)$

$= 63.64\,m.$

(b) New $v = \frac{120}{100} \times 25$

$= 30\,m/s$

Vertical

$s = ut - \frac{1}{2}gt^2$

$0 = 30 \sin 45 - \frac{1}{2}gt$

$\therefore t = 4.32\,s.$

Horizontal

$s = ut$

$= 30 \cos 45 (4.32)$

$= 91.64$

~~Reduction~~ =

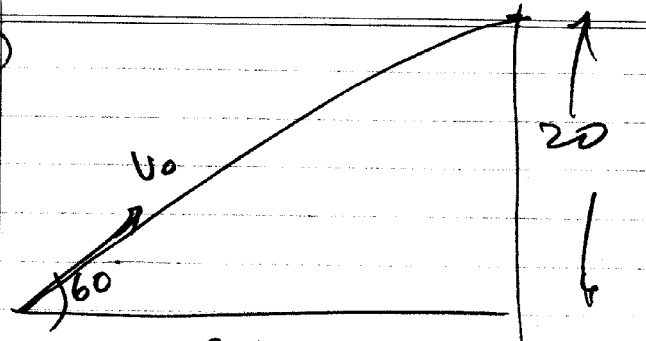
Increase $= 91.64 - 63.64$

$= 28$

% increase $= \frac{28}{63.64} \times 100$

$= 44\%$

(21)



(a) $t = 2.5$ s.

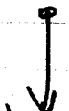
Vertical

$s = ut - \frac{1}{2}gt^2$

$20 = v_0 \sin 60 (2.5) - \frac{1}{2} g (2.5)^2$

$\therefore v_0 = 23.4$ m/s

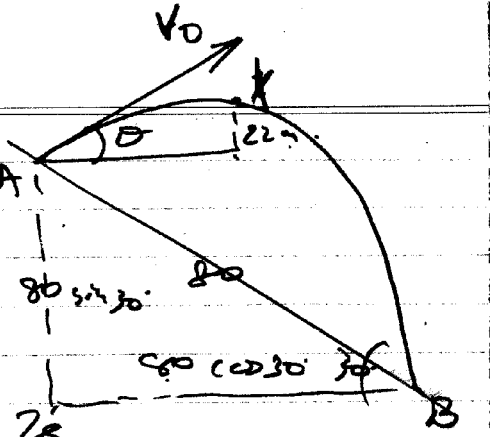
(b)



$s = ut + \frac{1}{2}gt^2$

$20 = 0 + \frac{1}{2} g (t^2) \quad \therefore t = 2.02$ s.

(22)



A - X

$v = 0$

$u = v_0 \sin\theta$

$s = 22$

$v^2 = u^2 - 2gs$

$0 = (v_0 \sin\theta)^2 - 2g(22)$

$\therefore v_0 \sin\theta = 20.78$ — (1)

A - B

Horizontal

$t = \frac{80 \cos 30}{v_0 \cos\theta}$

Vertical

$s = ut - \frac{1}{2}gt^2$

$-80\sin 30 = V_0\sin\theta\left(\frac{80\cos 30}{V_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{80\cos 30}{V_0\cos\theta}\right)^2$

From ① $V_0\sin\theta = 20.78$

$-80\sin 30 = \frac{20.78(80\cos 30)}{V_0\cos\theta} - \frac{1}{2}g\left(\frac{80\cos 30}{V_0\cos\theta}\right)^2$

$\therefore V_0\cos\theta = 12.21$ → ②

① ÷ ②

$\tan\theta = \frac{20.78}{12.21}$

$\therefore \theta = 59.56$

Sub in ①

$V_0 = 24.1$ m/s

(23) (i) Horizontal

$t = \frac{2}{V_0\cos 30°}$

Vertical

$s = ut - \frac{1}{2}gt^2$

$-1.5 = V_0\sin 30°\left(\frac{2}{V_0\cos 30°}\right) - \frac{1}{2}g\left(\frac{2}{V_0\cos 30}\right)^2$

$\therefore V_0 = 3.139$ m/s

(ii) Horizontal

$t = \frac{5}{V_0\cos 30°}$

Vertical

$s = ut - \frac{1}{2}gt^2$

$-1.5 = V_0 \sin 30 \left(\frac{5}{V_0 \cos 30}\right) - \frac{1}{2}g\left(\frac{5}{V_0 \cos 30}\right)^2$

$\therefore V_0 = 6.105$ m/s

$3.139 \leq V_0 \leq 6.105$ m/s

$\therefore$ m/s

(24)

$450 \sin 55°$

450

2500

$55°$

$450 \cos 55 + 6000 \times \frac{5}{18} =$ m/s

Horizontal $t = \frac{x}{424.78}$

Vertical

$s = ut - \frac{1}{2}gt^2$

$2500 = 450 \sin 53 \left(\frac{x}{424.78}\right) - \frac{1}{2}g\left(\frac{x}{424.78}\right)^2$

$x = 3202.1$ m

$\theta = \tan^{-1}\left(\frac{2500}{3202.1}\right) = 38°$

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